Paper Reference 4PM1/02
Pearson Edexcel
International GCSE

Further Pure Mathematics
PAPER 2
(Calculator)

Friday 7 June 2024 – Morning

Time: 2 hours

YOU MUST HAVE
Nil

YOU WILL BE GIVEN
Diagram Booklet
Formulae Pages
Answer Booklet

X76508A

Pearson

Calculators may be used.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions.

Without sufficient working, correct answers may be awarded no marks.

Answer the questions in the Answer Booklet or on the separate diagrams – there may be more space than you need.

Do NOT write on this Question Paper.

You must NOT write anything on the Formulae Pages. Anything you write on the Formulae Pages will gain NO credit.

INFORMATION

The total mark for this paper is 100.

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

There may be spare copies of some diagrams in case you need them.

You may be provided with models for Question 8 and Question 9
They are NOT accurate.

ADVICE

Read each question carefully before you start to answer it.

Check your answers if you have time at the end.

Answer all ELEVEN questions.

Write your answers in the Answer Booklet or on the separate diagrams.

You must write down all the stages in your working.

1. $f(x) = 6x^3 - 13x^2 + ax - 10$ where a is a constant

Given that

$(3x - 2)$ is a factor of $f(x)$

(a) show that

$a = 21$

(2 marks)

(b) Hence show algebraically that the curve $y = f(x)$ has only one intersection with the x–axis.

(4 marks)

(Total for Question 1 is 6 marks)

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2. The quadratic equation

$3x^2 - 5x + 1 = 0$ has roots α and β

Without solving the equation,

form a quadratic equation with integer coefficients,

that has roots

$\frac{\alpha}{2\beta}$ and $\frac{\beta}{2\alpha}$

(Total for Question 2 is 8 marks)

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3. Look at the diagram for Question 3 in the Diagram Booklet.

It is NOT accurately drawn.

It shows the sector AOB of a circle with centre O and radius $3r$ cm

A circle with radius r cm touches OA and OB and the arc AB

Angle AOB is θ radians, where

$$0 < \theta < \frac{\pi}{2}$$

(a) Find the exact value of θ

(2 marks)

The area of the region shown shaded in the diagram is 8π cm^2

(b) Find the value of r

(4 marks)

(Total for Question 3 is 6 marks)

4. Look at the diagram for Question 4 in the Diagram Booklet.

It shows part of the curve with equation

$$y = \frac{x^2}{3} - \frac{1}{2x} \text{ for } -4 < x < 0$$

By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation

$4x^3 + 3x^2 - 36x - 6 = 0$ in the interval

$-4 < x < 0$

(Total for Question 4 is 4 marks)

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5. $y = e^{2x}\left(x^2 - 5x\right)$

Show that $2e^{2x} = \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$

(Total for Question 5 is 7 marks)

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6. Look at the diagram for Question 6 in the Diagram Booklet.

It is NOT accurately drawn.

It shows part of the curve C with equation $y = \frac{1}{4x}$, $x > 0$ and part of the curve S with equation $y = 2x^2$, $x \geq 0$

The curve C and the curve S intersect at the point A

(a) Find the coordinates of point A

(3 marks)

The finite region R, shown shaded in the diagram, bounded by the curve C, the curve S and the straight line $y = 4$ is rotated through 360° about the y–axis.

(b) Find, using algebraic integration, the exact volume of the solid formed.

(7 marks)

(Total for Question 6 is 10 marks)

7. (a) Expand $\left(1 + 2x^2\right)^{-\frac{3}{4}}$ in ascending powers of x up to and including the term in x^6

Express each coefficient as an exact fraction in its lowest terms.

(3 marks)

$$f(x) = \frac{(2 + kx)}{\left(1 + 2x^2\right)^{\frac{3}{4}}} \text{ where } k \neq 0$$

(b) Obtain a series expansion for $f(x)$ in ascending powers of x up to and including the term in x^5

Give each coefficient in terms of k where appropriate.

(2 marks)

(continued on the next page)

7. continued.

The coefficient of the term in x^5 is fourteen times the coefficient of the term in x^2

(c) Find the value of k

(2 marks)

(Total for Question 7 is 7 marks)

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8. Look at Diagram 1, Diagram 2 and Diagram 3 for Question 8 in the Diagram Booklet.

You may be provided with a model.

They are NOT accurate.

Diagram 1 and the model show a 3D view of a solid right triangular prism $ABCDEF$

Diagram 2 shows the front face DCE

Diagram 3 shows the top face $ABCD$

The cross section of the prism is an isosceles triangle.

- angle DEC = angle $AFB = 90°$
- $AB = DC = x$ cm
- $AD = BC = FE = y$ cm
- $AF = BF = DE = CE$

The triangular faces of the prism are vertical and the edges AD, BC and FE are horizontal.

The volume of the prism is $3 \cdot 6$ cm^3

The total external surface area of the prism is S cm^2

(continued on the next page)

8. continued.

(a) Show that S satisfies the equation

$$S = \frac{x^2}{2} + \frac{72(\sqrt{2}+1)}{5x}$$

(4 marks)

Given that x can vary,

(b) use calculus, to find to 3 significant figures, the value of x for which S is a minimum.
Justify that this value of x gives a minimum value of S
(4 marks)

(c) Hence find, to 2 significant figures, the minimum value of S
(2 marks)

(Total for Question 8 is 10 marks)

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9. Look at Diagram 1, Diagram 2 and Diagram 3 for Question 9 in the Diagram Booklet.

You may be provided with a model.

They are NOT accurate.

Diagram 1 and the model show a 3D view of a right triangular prism **ABCDEF** where **ABCD** is a rectangle.

Diagram 2 shows the front face **AFB**

Diagram 3 shows the bottom face **ABCD**

AF = DE

BF = CE

AD = FE = BC

AB = DC = 24 cm

angle **ABF** = angle **DCE** = 45°

angle **BAF** = angle **CDE** = 60°

(continued on the next page)

9. continued.

Using a formula from the Formulae Pages,

(a) show that

$$\sin \mathrm{AFB} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

(3 marks)

Without using a calculator,

(b) show that

$$\mathrm{BF} = 12\left(3\sqrt{2} - \sqrt{6}\right) \text{ cm}$$

(5 marks)

(continued on the next page)

9. continued.

The angle between the plane AEB and the plane $ABCD$ is $65°$

(c) Find, in cm to 2 significant figures, the length of EF

(3 marks)

(d) Find, in degrees to one decimal place, the size of the angle between the line CF and the plane $ABCD$

(4 marks)

(Total for Question 9 is 15 marks)

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10. The points A, B, C and D are the vertices of a quadrilateral such that

$$\overrightarrow{AB} = 3\underline{a} + 4\underline{b}$$

$$\overrightarrow{AC} = 7\underline{a} + 9\underline{b}$$

$$\overrightarrow{AD} = 4\underline{a} + 5\underline{b}$$

(a) Show that $ABCD$ is a parallelogram.

(3 marks)

BC is extended to the point E such that BCE is a straight line.

Point F lies on CD such that $CF : FD = 1 : 2$

Given that A, F and E are collinear,

(b) find the vector $\overrightarrow{AE}$ in the form $X\underline{a} + Y\underline{b}$ where X and Y are rational numbers to be found.

(8 marks)

(Total for Question 10 is 11 marks)

11. Using formulae from the Formulae Pages, show that

(a) (i) $\cos 2A = 2\cos^2 A - 1$

(3 marks)

(ii) $\sin 2A = 2\sin A \cos A$

(1 mark)

(b) Show that

$$\cos^3 A = \frac{\cos 3A + 3\cos A}{4}$$

(4 marks)

(continued on the next page)

11. continued.

Hence, or otherwise,

(c) solve, giving exact values in terms of π

$$8\cos^3\left(\frac{\theta}{2}\right) - 6\cos\left(\frac{\theta}{2}\right) - 1 = 0 \qquad \text{for } 0 \leq \theta \leq 2\pi$$

(4 marks)

(d) use algebraic integration to find the exact value of

$$\int_0^{\frac{\pi}{6}} \left(4\cos^3\theta - \sin 2\theta\right) d\theta$$

(4 marks)

(Total for Question 11 is 16 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER